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Optimal Design of Large Structures for Damage Tolerance

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The basic idea presented herein is that a systematic method for optimal structural design can be developed that accounts for probable future damage to the structure. A general mathematical model for the damage-tolerant structure design problem is defined. Design sensitivity analysis and procedures for treatment of a large number of constraints for the problem are presented. As a practical design example, damage-tolerant design of an open truss helicopter tail-boom structure is considered. Optimal designs for several cases of the tail-boom are presented. It is shown that considerable design variation is necessary to achieve the damage-tolerant design objective. It is also shown that if the structure is optimized without consideration of damage, the structure will fail catastrophically when any damage occurs to the structure.

Nomenclature

b	= k vector of design variables
d	= total number of damage conditions
$K^{(\alpha)}(b)$	= $(n\times n)$ nonsingular symmetric stiffness matrix for the αth damage condition
$M^{(\alpha)}(b)$	= $(n\times n)$ mass matrix for the αth damage condition
$m(\alpha)$	= number of total constraints under the αth damage condition
$S^{(\alpha)}(b)$	= n vector of equivalent nodal loads for the αth damage condition
$y^{(\alpha)}$	= n eigenvector corresponding to the lowest eigenvalue $\zeta^{(\alpha)}$
$z^{(\alpha)}$	= n vector of nodal displacements, called the state variable vector of the finite element idealization for the αth damage condition
α	= superscript used to represent various quantities related to the αth damage condition
ψ_0	= cost function
$\psi_i^{(\alpha)}$	= ith constraint for the αth damage condition
$\zeta^{(\alpha)}$	= lowest eigenvalue for the αth damage condition

I. Introduction

MANY structures in civil, aerospace, and mechanical engineering are designed to minimize a measure of cost subject to various performance requirements. It is known from the theory of structural optimization that several of the performance constraints for the structure are at their limit values for its optimum performance. Thus, a slight variation in the performance environment for the structure or minor imperfections in its fabrication can cause severe violation of various performance constraints. This behavior can cause catastrophic failure of the structure. Thus, the recent trend toward optimal design of structures has alarmed many designers regarding the safety of the optimized structure.

In addition, many structures are required to survive in hazardous environments. These environments are characterized by rarely occurring, high intensity, short duration loads. It may be impractical or uneconomical to design a structure that is unaffected by such loads. Thus, a certain amount of damage is bound to occur during the performance life of the structure.

A basic hypothesis of this paper is that a structure can be designed so that it is invulnerable to some future damage. The idea is to anticipate the future damage to the structure and account for it in the design process. The following definitions relative to this design problem are presented:

1) Damage-tolerant structure. A structure is called damage-tolerant or fail-safe if it continues to perform its basic function even after it sustains a specified level of damage.

2) Damage condition. A damage condition for the structure is defined to consist of complete or partial removal of selected members or parts of the structure. Some joints of the structure may be removed as a result of the damage.

3) Optimal damage-tolerant structure. A damage-tolerant structure is called optimal if it is designed to minimize a cost function, subject to constraints that must hold for the undamaged structure and under projected damage conditions.

A basic assumption in the proposed design method is that the structure remains geometrically stable after some damage to its members or joints. In other words, the structure does not fail catastrophically in a mechanism-type motion after damage occurs. The structure is generally assumed to have enough redundancy in its construction.

A comprehensive review of literature related to damage-tolerant design of structures was conducted in Ref. 1. No significant literature was found related to this class of problems. Also, a preliminary method for optimal design of damage-tolerant structures was presented in Ref. 1. The method was applied to small-scale structures. That method is generalized here and is used to design an open truss helicopter tail-boom.

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A general mathematical model for damage-tolerant design of linearly elastic structures is presented in Sec. II. A finite-element model of the structure is employed to obtain its static and frequency response. A state space method of design sensitivity analysis (calculation of gradients or constraints with respect to design variables) that has been shown to be quite general and efficient in Refs. 2 and 3 is presented in Sec. III, and an optimization algorithm for design of damage-tolerant structures is discussed. The algorithm is an extension of the gradient projection method applied to mechanical and structural design problems.[4-6] In Sec. IV, formulation of a practical problem of damage-tolerant design of an open truss helicopter tail-boom is presented. This problem has 546 system variables and approximately 1600 constraints. The structure is designed to withstand one loading condition and six projected damage conditions. Finally, Sec. V presents a discussion of the method and general conclusions based on the study.

II. Mathematical Model for Damage-Tolerant Optimal Structural Design

A mathematical model for damage-tolerant optimal structural design is defined as follows: Find a design variable vector $b \in R^k$ that minimizes a cost function

$$\psi_0(b,z^{(\alpha)},\zeta^{(\alpha)}) \tag{1}$$

while satisfying the governing equations of the finite element idealization for the structure,

$$K^{(\alpha)}(b)z^{(\alpha)} = S^{(\alpha)}(b) \tag{2}$$

$$K^{(\alpha)}(b)y^{(\alpha)} = \zeta^{(\alpha)}M^{(\alpha)}(b)y^{(\alpha)} \tag{3}$$

and the constraints

$$\psi_i^{(\alpha)}(b,z^{(\alpha)},\zeta^{(\alpha)}) \leq 0, \quad i=1,2,\ldots,m(\alpha)$$
$$\alpha = 0,1,2,\ldots,d \tag{4}$$

Note that $\alpha=0$ represents quantities relative to the undamaged structure. The cost function of Eq. (1) is quite general and may represent weight (mass) of the structure, displacement at some points, or an eigenvalue related cost function for a damaged or undamaged structure. The load vector $S^{(\alpha)}(b)$ may be expanded to a matrix to account for multiple loading conditions. The applied loads depend upon the parameter α, implying that different load-bearing functions may be specified for the undamaged and the damaged structures. Also the load vector is taken as a function of the design variable vector b to account for the effect of body forces and thermal loads. The eigenvalue problem of Eq. (3) may govern the free vibration response or overall buckling behavior of a damaged or undamaged structure. Constraints of Eq. (4) are also quite general. They represent stress and displacement constraints under multiple loading and displacement conditions, eigenvalue bounds for undamaged and damaged structures, and explicit bounds on member sizes to account for practical design considerations. It should also be noted that no assumption is made on the type of finite-elements to be used in modeling the structure. Thus, the above mathematical model for damage-tolerant optimal design of structural systems is quite general, as it can treat a wide variety of design problems.

III. Design Sensitivity Analysis and Optimization Method

In most iterative optimal design methods, one needs to calculate explicit derivatives of the cost and constraint functions with respect to the design variables. Several methods for computing these derivatives are available.[2] A method that has been shown to be quite general and efficient[2,3] is summarized here for the damage-tolerant optimal structural design problem.

Since the form of cost and constraint functions are similar, a typical function $\psi_i^{(\alpha)}(b,z^{(\alpha)},\zeta^{(\alpha)})$ is considered for calculation of derivatives with respect to the design variables. A first variation of the functions $\psi_i^{(\alpha)}$, treating $b, z^{(\alpha)}$, and $\zeta^{(\alpha)}$ as independent variables, is given as

$$\delta\psi_i^{(\alpha)} = \frac{\partial\psi_i^{(\alpha)}}{\partial b}\delta b + \frac{\partial\psi_i^{(\alpha)}}{\partial z^{(\alpha)}}\delta z^{(\alpha)} + \frac{\partial\psi_i^{(\alpha)}}{\partial\zeta^{(\alpha)}}\delta\zeta^{(\alpha)} \tag{5}$$

Here explicit arguments $b, z^{(\alpha)}, \zeta^{(\alpha)}$ of the function $\psi_i^{(\alpha)}$ are omitted for convenience; the partial derivatives $\partial\psi_i^{(\alpha)}/\partial b$, $\partial\psi_i^{(\alpha)}/\partial z^{(\alpha)}$, and $\partial\psi_i^{(\alpha)}/\partial\zeta^{(\alpha)}$ are calculated at the given value of b and the calculated values of $z^{(\alpha)}$ and $\zeta^{(\alpha)}$; and δb, $\delta z^{(\alpha)}$, and $\delta\zeta^{(\alpha)}$ represent first variations in the variables b, $z^{(\alpha)}$, and $\zeta^{(\alpha)}$, respectively. It is clear that any variation δb in the design b induces variations in the variables $z^{(\alpha)}$ and $\zeta^{(\alpha)}$. If one can express terms related to $\delta z^{(\alpha)}$ and $\delta\zeta^{(\alpha)}$ in terms of δb in Eq. (5), then one may write Eq. (5) as

$$\delta\psi_i^{(\alpha)} = \Lambda^{i(\alpha)T}\delta b \tag{6}$$

where $\Lambda^{i(\alpha)}$ is the desired gradient vector with respect to the design variable. The procedure of Refs. 2-6 does just that.

First, a variation of the general eigenvalue problem of Eq. (3) yields the following expression for $\delta\zeta^{(\alpha)}$, after some algebraic manipulations[4]:

$$\delta\zeta^{(\alpha)} = \omega^{(\alpha)T}\delta b \tag{7}$$

where

$$\omega^{(\alpha)} = \frac{\partial}{\partial b}\left[K^{(\alpha)}(b)y^{(\alpha)} - \zeta^{(\alpha)}M^{(\alpha)}(b)y^{(\alpha)}\right]^T y^{(\alpha)}/M^*$$
$$M^* = y^{(\alpha)T}M^{(\alpha)}(b)y^{(\alpha)} \tag{8}$$

and $M^* \neq 0$, since $y^{(\alpha)}$ is an eigenvector of Eq. (3). Thus, substitution of $\delta\zeta^{(\alpha)}$ from Eq. (7) into Eq. (5) yields

$$\delta\psi_i^{(\alpha)} = \left[\frac{\partial\psi_i^{(\alpha)}}{\partial b} + \frac{\partial\psi_i^{(\alpha)}}{\partial\zeta^{(\alpha)}}\omega^{(\alpha)T}\right]\delta b + \frac{\partial\psi_i^{(\alpha)}}{\partial z^{(\alpha)}}\delta z^{(\alpha)} \tag{9}$$

A first variation of the equilibrium equation, Eq. (2) gives

$$K^{(\alpha)}(b)\delta z^{(\alpha)} = \frac{\partial}{\partial b}\left[S^{(\alpha)}(b) - K^{(\alpha)}(b)z^{(\alpha)}\right]\delta b \tag{10}$$

Now substituting for $\delta z^{(\alpha)}$ [after inverting $K^{(\alpha)}(b)$] from Eq. (10) into Eq. (9) and introducing the notation

$$\frac{\partial\psi_i^{(\alpha)}}{\partial z^{(\alpha)}}[K^{(\alpha)}]^{-1} = \lambda^{i(\alpha)T} \tag{11}$$

one obtains Eq. (6), where the gradient vector $\Lambda^{i(\alpha)}$ is given as

$$\Lambda^{i(\alpha)} = \frac{\partial\psi_i^{(\alpha)T}}{\partial b} + \omega^{(\alpha)}\frac{\partial\psi_i^{(\alpha)}}{\partial\zeta^{(\alpha)}}$$
$$+ \frac{\partial}{\partial b}\left[S^{(\alpha)}(b) - K^{(\alpha)}(b)z^{(\alpha)}\right]^T\lambda^{i(\alpha)} \tag{12}$$

The vector $\lambda^{i(\alpha)}$ is called an adjoint vector and is obtained from Eq. (11). Post-multiplying Eq. (11) by $K^{(\alpha)}$, taking transpose, and using the fact that $K^{(\alpha)T} = K^{(\alpha)}$, one obtains the linear equation for $\lambda^{i(\alpha)}$ as

$$K^{(\alpha)}\lambda^{i(\alpha)} = \frac{\partial \psi_i^{(\alpha)T}}{\partial z^{(\alpha)}} \tag{13}$$

Equation (13) is the same as Eq. (2) with a different right-hand side. Thus, decomposed $K^{(\alpha)}(b)$ is used readily to calculate the adjoint variable vector $\lambda^{i(\alpha)}$ efficiently.

Note that calculations of $\partial\psi_i^{(\alpha)}/\partial b$, $\partial\psi_i^{(\alpha)}/\partial z^{(\alpha)}$, and $\partial\psi_i^{(\alpha)}/\partial\zeta^{(\alpha)}$ are performed readily once the form of the function $\psi_i^{(\alpha)}$ is determined. Also calculations of $\partial S^{(\alpha)}(b)/\partial b$, $\partial/\partial b[K^{(\alpha)}(b)z^{(\alpha)}]$, and $\omega^{(\alpha)}$ are performed readily and efficiently by considering contributions from each member of the finite element idealization of the structure.[5,6]

After design sensitivity analysis of the damage-tolerant structural design problem has been completed, any optimization algorithm may be used to compute the optimum solution. The authors and their coworkers have successfully used a generalized steepest descent method on a wide range of structural and mechanical design problems.[1-6] Therefore, this method is used for the damage-tolerant structural design problem.

One may view the generalized steepest descent method as a gradient projection approach in design space. With this viewpoint, one simply computes constraint boundaries associated with the performance requirements on the structure. The tangent hyperplanes of active constraints are then included in the gradient projection calculation. Modification to treat constraints on damaged configurations requires only a minor adjustment to the basic gradient projection algorithm of Refs. 4-6. It requires only that the damaged conditions be analyzed, adjoint equations associated with active constraints be solved, and design sensitivity vectors be included in the gradient projection calculations. The algorithm is stated in Refs. 1 and 6.

An important consideration relative to damage-tolerant optimal design is the selection of critical constraints in any design iteration. The damage-tolerant optimal design problem is characterized by requiring the design variables to satisfy constraints whose number is much larger than the number of design variables. According to the theory of simultaneous equations, there can be exactly as many independent active constraints as the number of design variables, in order to determine the design variables uniquely. If there are more active constraints than the number of design variables, the set of sensitivity vectors $[\tilde{\Lambda}^i]$ is linearly dependent. This causes difficulty in calculating the Lagrange multipliers that are needed in implementing the algorithm.[7] Thus, if all active constraints are considered at any design iteration, much computation is required and the accuracy of the result may be jeopardized. A rational way to select an independent critical constraint set is essential.

A linearized constraint

$$\tilde{\psi}_i + \tilde{\Lambda}^{iT}\delta b \le 0 \tag{14}$$

taken as an equality, represents a hyperplane in δb space. The distance from this hyperplane to the current design point is

$$D_i = |\tilde{\psi}_i| / \|\tilde{\Lambda}^i\| \tag{15}$$

where $\|\tilde{\Lambda}^i\|$ is the norm of the sensitivity vector.

The distance from a constraint hyperplane to the current design point is a good measure of the criticality of the constraint. It is clear that, among a parallel set of active constraints, the one most remote from the current design point is the critical constraint. For a set of nearly parallel constraints, when the design change is small, the furthest few constraints are most critical. For stress constraints on a member, the partial derivatives assocaited with the design variable of this member usually dominates.[8] Thus, the stress constraints on a member under different loading conditions and damage consitions can be considered to be a parallel set of constraints and only the worst violated one needs to be considered.

From the preceding discussion, the following strategy can be used in selecting critical stress constraints for weight minimizing problems. For a given member, only the worst violated stress constraint over all damage conditions is considered. When all constraints are checked and sensitivity vectors are constructed, these constraints are compared according to their distances from the current design point, keeping only the furthest independent constraints.

There are other computational considerations for efficient implementation of the method. It is clear that if a set of data is needed more than once, storing it may take less computer time than reconstructing it every time it is needed. For small problems this is possible, but for larger problems the storage required can easily exceed the capacity of the large-scale computer. Thus, only frequently used information should be stored.

For a given damage condition, the coefficient matrices for the state equation (2) and for the adjoint equation (13) are the same. They cannot be solved at the same time, because the state equations must be solved before checking constraints. Thus, the decomposed stiffness matrix must either be stored or reconstructed as many times as needed. Several authors have developed methods of finding displacements of a slightly modified structure,[9] either by modifying the stiffness matrix or its decomposed factors $K = U^TU$ or by modifying displacements. These procedures may also be incorporated in the method to enhance its computational efficiency.

IV. Damage-Tolerant Design of an Open Truss Helicopter Tail-Boom

In order to evaluate the fail-safe design algorithm, a moderate size, realistic structure that can be modeled as a truss is chosen. The structure is the tail-boom for a helicopter. The basic configuration and end sections of the tail-boom are shown in Fig. 1. The maximum in-flight loads to be supported by the tail-boom structure are also shown in Fig. 1. The structure that is currently in use consists of longitudinal members, cross members, and a skin cover to obtain an enclosed tail-boom. This type of structure is vulnerable to blasts that occur inside or near the skin. In order to reduce vulnerability of the structure to such damage, an open truss type structure is considered. Accordingly, the structure shown in Fig. 1 is modeled as a 108-member truss with 28 joints and

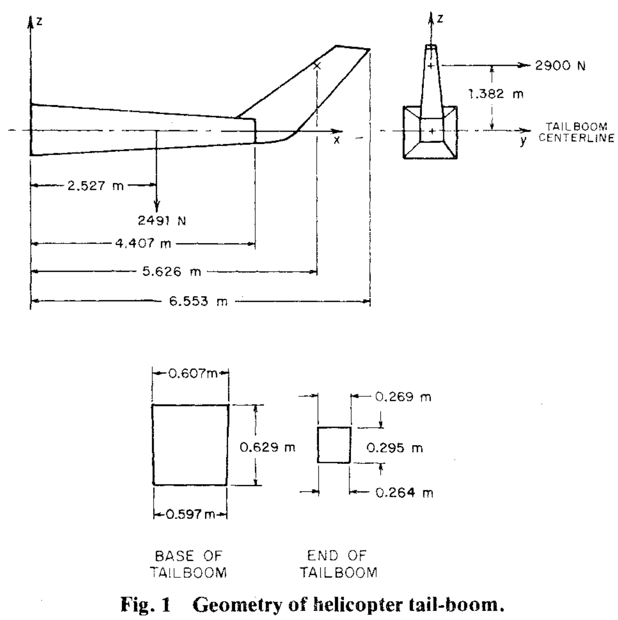


Fig. 1 Geometry of helicopter tail-boom.

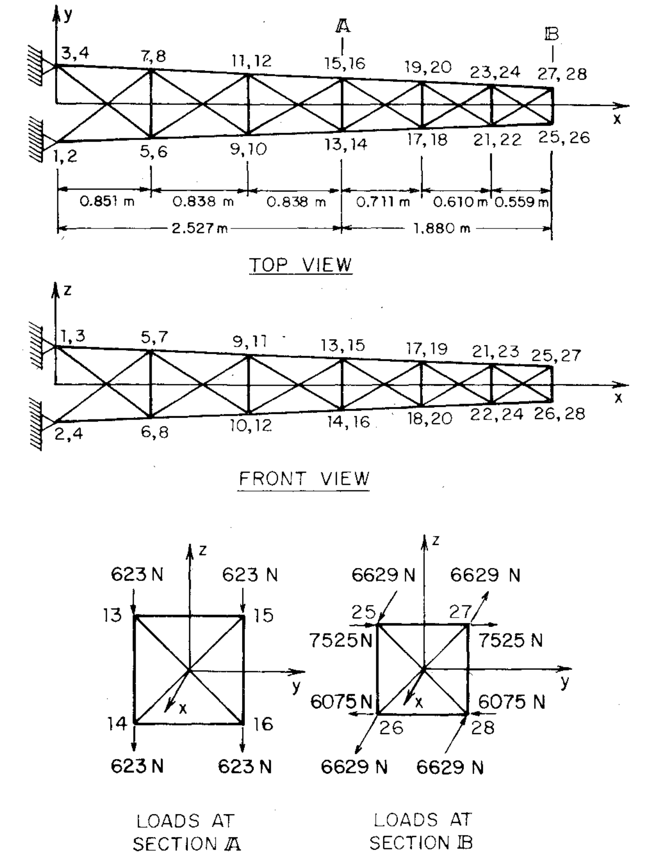
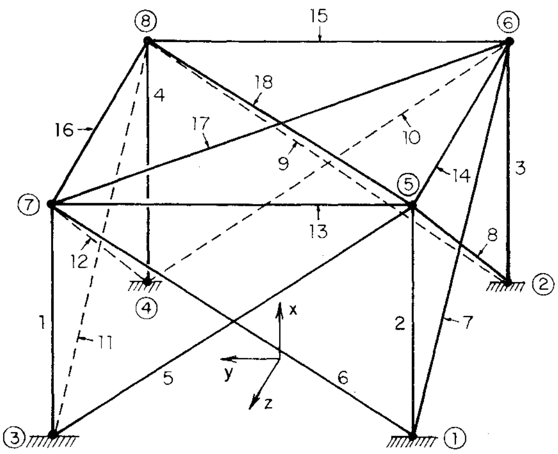


Fig. 2 Arrangement of members for open truss helicopter tail-boom.



Following grouping of members, with members of a group to have same cross-sectional areas is used:

Group No.	Member Numbers
1	2, 3
2	1, 4
3	5, 6, 9, 10
4	7, 8, 11, 12
5	13, 15
6	14, 16
7	17, 18

Fig. 3 Member numbering for first panel.

72 deg of freedom. The geometry of the idealized structure and the design loads are given in Fig. 2. The element numbering system for a typical panel is shown in Fig. 3. The member definitions for the structure are given in Table 1.

Table 1 Member locations for open truss helicopter tail-boom

Member	End nodes	Member	End nodes	Member	End nodes
1	3 7	37	11 15	73	19 23
2	1 5	38	9 13	74	17 21
3	2 6	39	10 14	75	18 22
4	4 8	40	12 16	76	20 24
5	3 5	41	11 13	77	19 21
6	1 7	42	9 15	78	17 23
7	1 6	43	9 14	79	17 22
8	2 5	44	10 13	80	18 21
9	2 8	45	10 16	81	18 24
10	4 6	46	12 14	82	20 22
11	3 8	47	11 16	83	19 24
12	4 7	48	12 15	84	20 23
13	7 5	49	15 13	85	23 21
14	5 6	50	13 14	86	21 22
15	6 8	51	14 16	87	22 24
16	8 7	52	16 15	88	24 23
17	7 6	53	15 14	89	23 22
18	8 5	54	16 13	90	24 21
19	7 11	55	15 19	91	23 27
20	5 9	56	13 17	92	21 25
21	6 10	57	14 18	93	22 26
22	8 12	58	16 20	94	24 28
23	7 9	59	15 17	95	23 25
24	5 11	60	13 19	96	21 27
25	5 10	61	13 18	97	21 26
26	6 9	62	14 17	98	22 25
27	6 12	63	14 20	99	22 28
28	8 10	64	16 18	100	24 26
29	7 12	65	15 20	101	23 28
30	8 11	66	16 19	102	24 27
31	11 9	67	19 17	103	27 25
32	9 10	68	17 18	104	25 26
33	10 12	69	18 20	105	26 28
34	12 11	70	20 19	106	28 27
35	11 10	71	19 18	107	27 26
36	12 9	72	20 17	108	28 25

The problem is to minimize the total mass of the structure and at the same time to ensure that member stress, nodal displacement, member buckling, and natural frequency constraints are satisfied under projected loading and damage conditions. The design parameters to be calculated are the cross-sectional areas of the members. A lower bound constraint is also imposed on cross-sectional area.

The members of the truss are taken to be tubular sections. Assuming the sections to be thin, the moment of inertia and cross-sectional area are given as $I = \pi R^3 t$ and $A = 2\pi R t$, where R is the mean radius and t is the thickness of the tube. In calculating the Euler buckling load, the moment of inertia is assumed to be given as $I = \beta A^2$. Therefore, $\beta = I/A^2$ is given as $R/4\pi t$. If R/t is conservatively selected as 12-14, then $\beta \simeq 1.0$. This value of β is used in calculations.

Design data for the structure are given in Table 2. The working stress for each member is assumed to be approximately 60% (± 172 MPa) of the yield stress (289 MPa) for the material used. This working stress corresponds to a safety factor of roughly 1.68. Displacement limit of ± 1.27 cm at the nodal points are based on approximately 1/3 deg misalignment at the center of the tail-boom. The lower limit on member cross-sectional area is taken as 0.2677 cm^2 which corresponds to a tube with 1.27 cm outside diameter and 0.071 cm wall thickness. There is no upper limit on cross-sectional area. There is only one loading condition for the structure, which is given in Table 2. There are six projected damage conditions for the structure, given in Table 3 and Fig. 4. For each damage condition a joint of the structure and all members connected to the joint are removed. Thus, each damaged structure has different stiffness and mass matrices and state variables. Note, however, that each damaged structure is geometrically stable.

Table 2 Design data for open truss helicopter tail-boom

Data common to both undamaged and damaged structures:	
Material	2024-T3 aluminum alloy
Modulus of elasticity	72.4 GPa
Stress limits	±172 MPa
Material density	2.77×10^3 kg/m^3
Moment of inertia	$I = \beta A^2$, $\beta = 1.0$
Displacement limits	1.27 cm
Lower limit on cross-sectional area	0.2677 cm^2
Upper limit on cross-sectional area	None
Lower bound on natural frequency for undamaged structure	29 Hz

Loading data for undamaged structure[a]:

Node number	Load component (*N*), in direction: *X*	*Y*	*Z*
13	0.0	0.0	−623
14	0.0	0.0	−623
15	0.0	0.0	−623
16	0.0	0.0	−623
25	6629	7525	0.0
26	6629	−6075	0.0
27	−6629	7525	0.0
28	−6629	−6075	0.0

[a] Number of loading conditions = 1.

Table 3 Damage condition definitions

Damage condition	Members damaged	Node damaged	Reduction in area, %
1	21,25,28,32,33, 35,39,44,45	10	100
2	1,6,12,13,16, 17,19,23,29	7	100
3	58,63,65,69,70, 72,76,82,84	20	100
4	73,78,84,85,88, 89,91,95,101	23	100
5	56,59,62,67,68, 72,74,78,79	17	100
6	3,7,10,14,15, 17,21,26,27	6	100

In order to maintain symmetry and to facilitate fabrication of the structure, 108 members of the structure are divided into a total of 42 groups and each group is assigned a design variable. Therefore, each panel of the structure (shown in Fig. 3) has seven design variables. Also, it is interesting to study the effect on structural weight obtained by imposing varying degrees of performance requirements for the damaged structures. Thus, optimum solutions for the following five cases are obtained:

Case I: Complete structure with no damage.

Case II: Complete structure with damage conditions 1-6 imposed and the structural load and natural frequency requirements for damaged structures reduced to two-thirds of the normal conditions.

Case III: Same as case II except load and natural frequency requirements for damaged structures are 80% of the normal conditions.

Case IV: Same as case II except load and natural frequency requirements for damaged structures are 90% of the normal conditions.

Case V: Complete and damaged structures required to perform for full set of normal conditions.

Optimum designs for the open truss helicopter tail-boom for cases I-V are given in Table 4. These designs were obtained by starting the iterative process with 6.45 cm^2 as cross-sectional area for all members of the tail-boom. Comparing the results for cases I and II, one concludes that when performance requirements for projected damaged structures (defined in Table 3) are reduced to two-thirds of the normal conditions there is essentially no penalty on the mass of the structure. However, there is some redistribution of the material, as may be seen from optimal solutions for cases I and II given in Table 4. If the final design for case I given in Table 4 is taken as the starting design for case II, there are large constraint violations. This indicates that the structure constructed from the solution of case I would fail catastrophically if any of the damage conditions defined in Table 3 occurred, even after the load and the natural frequency requirements were reduced to two-thirds of the normal conditions. On the other hand, if a tail-boom is constructed from the final areas for case II, the structure is able to support safely two-thirds of the load carrying requirement, even after any of the specified damage occurs.

Final designs for cases III-V are also given in Table 4. They indicate that there is a substantial penalty on the mass of the structure as the load carrying and natural frequency requirements for the damaged structures are increased.

Due to ease in fabrication, it is desirable to use as few standard sections as possible. For the design of cases I-V, 42 design variables (that is, 42 types of sections) are used. This number is perhaps too large. Therefore tail-boom design for two additional cases VI and VII are also obtained. These cases are as follows:

Case VI: The number of design variables is reduced to 12, with 2 design variables for each bay. For the first bay of Fig. 3, members 1-4 and 13-16 have the same cross-sectional areas and members 5-12, 17 and 18 have the same cross-sectional areas. The tail boom is designed with six damage conditions of Table 3 imposed, and complete and damaged structures are required to perform for full set of normal conditions.

Case VII: The number of design variables is reduced to four with two design variables for first three bay and two design variables for the last three bays. For the first three bays, all longerons, vertical, and cross-members have same cross-sectional areas and all diagonals have the same cross-sectional areas. A similar grouping is done for the last three bays. The tail-boom is designed with six damage conditions of Table 3 imposed, and complete and damaged structures are required to perform for full set of normal conditions.

The optimal designs for the last two cases are also obtained using the same computer code[10] and by starting from uniform cross-sectional areas of 6.45 cm^2 for all members. The final areas for case VI are given in Table 5 and for the case VII in Table 6. As expected, there is a substantial penalty on mass of the structure, as computed to the mass obtained in case V. This indicates that the designer has to decide whether the mass of the structure or its fabrication cost is critical, because as the number of design variables is reduced the optimum mass of the structure increases.

The constraints that are critical at the optimum for all cases are given in Table 7. For all cases, all active constraints are

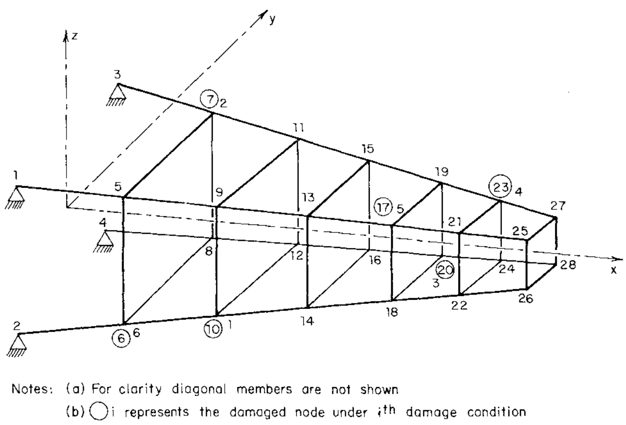


Fig. 4 Nodes damaged under various damage conditions.

Table 4 Optimum designs for cases I-V of the tail-boom structure

		Final cross-sectional areas, cm^2				
Design variable	Member numbers	Case I	Case II	Case III	Case IV	Case V
1	2, 3	8.8709	9.1290	10.923	14.716	19.516
2	1, 4	8.8451	9.1871	10.606	13.690	17.987
3	5, 6, 9, 10	0.8871	0.8974	1.3510	1.5658	1.7316
4	7, 8, 11, 12	0.8999	0.9961	0.9445	1.0252	1.4619
5	13, 15	0.2677	0.2677	0.2677	0.4684	0.6439
6	14, 16	0.5297	0.5219	0.8864	1.0968	1.0252
7	17, 18	0.2677	0.2677	1.1464	2.0561	2.0439
8	20, 21	8.0129	8.1355	8.9484	11.006	14.477
9	19, 22	7.9935	8.1290	7.9999	10.174	13.503
10	23, 24, 27, 28	1.1232	1.0277	1.1297	1.3942	2.5574
11	25, 26, 29, 30	1.0639	1.2026	2.2606	2.5684	2.7226
12	31, 33	0.2677	0.2677	0.3090	0.2677	0.3077
13	32, 34	0.6465	0.6671	1.2568	1.2723	0.9819
14	35, 36	0.2677	0.3213	0.5865	0.6677	0.7942
15	38, 39	6.6387	6.5935	6.8064	7.1355	8.4258
16	37, 40	6.6322	6.4968	6.4774	6.9032	8.1935
17	41, 42, 45, 46	1.3613	1.2839	1.4845	1.6677	2.1961
18	43, 44, 47, 48	1.4806	1.6213	1.5897	1.7664	1.8671
19	49, 51	0.2677	0.2677	0.3213	0.5777	0.5987
20	50, 52	0.8845	0.8484	0.7923	0.6206	0.5799
21	53, 54	0.2677	0.2677	0.2910	0.4987	0.5787
22	56, 57	5.3039	5.3019	5.3142	5.6510	6.0083
23	55, 58	5.3071	5.1742	5.2768	5.8348	6.2974
24	59, 60, 63, 64	1.5258	1.4942	1.9645	2.4084	2.6084
25	61, 62, 65, 66	1.6690	1.5645	1.2199	0.9729	1.0213
26	67, 69	0.2677	0.2677	0.2677	0.4858	0.5819
27	68, 70	1.0161	0.8852	1.1064	0.7935	0.6955
28	71, 72	0.2677	0.3245	0.8277	1.1258	1.1058
29	74, 75	3.7458	3.7716	3.2226	3.4774	3.5581
30	73, 76	3.7613	3.6703	3.6297	4.2793	4.3006
31	77, 78, 81, 82	1.7258	1.6942	1.5039	1.7103	1.8929
32	79, 80, 83, 84	1.8600	1.7387	2.2277	2.1116	2.0071
33	85, 87	0.2677	0.2677	0.2897	0.2684	0.3742
34	86, 88	1.2477	1.0813	1.3755	1.1000	1.0148
35	89, 90	0.2677	0.2677	0.3510	0.6897	0.7890
36	92, 93	1.4832	1.4477	1.4671	1.7303	1.7677
37	91, 94	0.3484	1.4516	1.3039	0.8852	0.7639
38	95, 96, 99, 100	2.1258	2.0568	1.8742	1.3768	1.1968
39	97, 98, 101, 102	2.2116	2.0955	2.1406	2.1819	2.1464
40	103, 105	0.3689	0.2677	0.4884	0.5942	0.7026
41	104, 106	0.6684	0.5645	0.6445	0.6110	0.5974
42	107, 108	1.2819	1.2445	1.2290	1.2252	1.1755
Mass, kg		47.9	48.0	53.0	61.2	73.1
Average CPU/iterative on IBM 370-158 (G), s		4.0	24.0	26.4	26.7	26.7
Number of active constraints at optimum		12	14	11	14	10

Table 5 Optimum design for case VI of helicopter tail-boom

Design variable	Member numbers	Final areas, cm^2
1	1-4, 13-16	18.948
2	5-12, 17, 18	3.6761
3	19-22, 31-34	13.181
4	23-30, 35-36	5.4574
5	37-40, 49-52	6.9419
6	41-48, 53, 54	2.6110
7	55-58, 67-70	4.5374
8	59-66, 71, 72	2.3323
9	73-76, 85-88	2.8839
10	77-84, 89, 90	2.1252
11	91-94, 103-106	1.0026
12	95-102, 107-108	1.6200
Mass, kg		109.6
Average CPU/cycle on IBM 370-158 (G),s		26.8
Number of active constraints at optimum		5

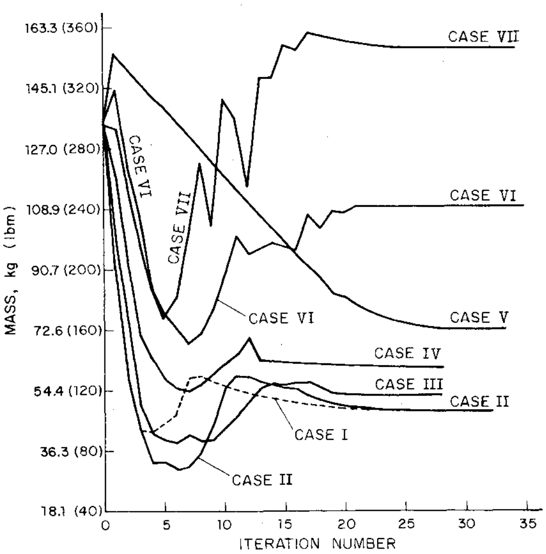


Fig. 5 Cost function histories for several design cases of helicopter tail-boom truss.

Table 6 Optimum design for case VII of helicopter tail-boom

Design variable	Member numbers	Final areas, cm^2
1	1-4, 13-16, 19-22, 31-34, 37-40, 49-52	21.264
2	5-12, 17, 18, 23-30, 35, 36 41-48, 53, 54	5.7387
3	55-58, 67-70, 73-76, 85-88, 91-94, 103-106	2.7632
4	59-66, 71, 72, 77-84, 89, 90 95-102, 107, 108	1.8039
Mass, kg		157.1
Average CPU/cycle on IBM 370-158 (G), s		18.0
Number of active constraints at optimum		3

Table 7 Critical constraints at optimum

Case I: Displacement in the y direction at nodes 25 and 27, and lower limit on design variable numbers 5, 7, 12, 14, 19, 21, 26, 28, 33, and 35.

Case II: Same as in case I, except design variables 14 and 28 are not at their lower bounds and 40 is at its lower bound, and buckling constraint for members 18, 36, 71 are tight under damage conditions 6, 1, and 5, respectively.

Case III: Displacement in the y direction at node 25 under damage conditions 1, 2, 4, 5, and 6; displacement in the y direction at node 27 under damage conditions 1, 2, 5, and 6; and lower bound on design variables 5 and 26.

Case IV: Displacement in the y direction at node 25 under damage conditions 1, 2, 3, 4, and 5; displacement in the y direction at node 27 under damage conditions, 1, 2, 3, and 5; frequency constraints under damage conditions 2 and 6; buckling constraint for member 66 under damage condition 3; and lower bound on design variables 12 and 33.

Case V: Displacement in the y direction at node 25 under damage conditions 2, 3, 4, and 5; displacement in the y direction at node 27 under damage conditions 2, 3, and 5; frequency constraints under damage conditions 2 and 6; buckling constraint for member 66 under damage condition 3.

Case VI: Displacement in the y direction at nodes 25 and 27 under damage conditions 4 and 5; and frequency under damage condition 2.

Case VII: Displacement in the y direction at nodes 25 and 27 under damage condition 5; and frequency constraint under damage condition 2.

satisfied to within 0.10% of their allowable values. The natural frequencies of the complete and damaged structures at the optimum solution are given in Table 8. The cost function histories for all cases are given in Fig. 5. In most cases, an optimum design or a design very close to the optimum was obtained in 20-30 iterations.

The rate of convergence to the optimum is highly dependent upon proper selection of a step size parameter. In order to see how critical the step size parameter is, several step sizes for case II of the helicopter tail-boom were tried and it was possible to obtain convergence to the optimum in 20 iterations, as compared to 32 iterations as shown in Fig. 5. The step size in all calculations were selected based upon the idea of specifying a desired reduction in the cost function.[4-6] For more discussion on step-size selection, Ref. 7 may also be consulted.

It should be noted that in the first few design iterations for all cases of the tail-boom design, a large number of constraints (50-100) were violated. The maximum violation was of the order of 1500% of the nominal value. The gradient projection method for optimal structural design corrected these constraint violations without any difficulty.

V. Discussions and Conclusions

In the paper, a general mathematical model for optimal design of damage-tolerant structures is defined. An efficient design sensitivity analysis procedure for the problem is presented and integrated into the gradient projection method.

As a preliminary application of the method, optimal design of an open truss helicopter tail-boom is considered. Optimum designs for several cases of the tail-boom are obtained and the results analyzed. The following conclusions are based on this study:

1) A structure that is optimized without imposing damage conditions will generally fail catastrophically if any damage occurs to the structure at a later date. A significant design variation is required to achieve damage-tolerant design objectives.

2) Comparison of results for case V-VII shows that the mass of the structure increases substantially as the number of design variables, that is, the number of sections from which the structure is to be fabricated, is reduced. Thus, there is a trade-off between optimum structural mass and fabrication costs. This is an important practical design consideration.

3) The computational algorithm performed well for the tail-boom design problem.

4) Computational time was reasonable for this moderate-size structure.

5) Considerable analytical and numerical work is required to realize full potential of the method.

Finally, it is noted that work is in progress[11] to incorporate substructuring in design sensitivity analysis and the optimization algorithm. Also, the finite element library is being

Table 8 Structural frequency at optimum

Damaged condition	Frequency at optimum, Hz						
	Case I	Case II	Case III	Case IV	Case V	Case VI	Case VII
0[a]	34.34	34.90	36.75	39.80	44.12	42.52	43.19
1	—	24.83	26.00	27.44	30.85	31.79	35.08
2	—	22.06	23.82	26.10	29.00	29.00	29.00
3	—	35.61	37.58	40.81	44.70	43.56	41.55
4	—	37.62	39.64	42.81	47.21	45.81	45.77
5	—	35.52	37.38	40.53	44.40	43.46	41.36
6	—	22.42	23.85	26.10	29.00	29.41	29.42

[a] Undamaged structure.

expanded to include constant strain triangular and bending elements so that more complex structural systems may be treated by damage-tolerant optimal design algorithm.

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